

Concern Regarding “A Non-Inflationary Solution to the Entropy Problem of Standard Cosmology”.

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We discuss the entropy and the size/homogeneity/horizon problems in power-law expanding universes with one scale initial conditions. We set the minimal scale $\Lambda = 10^{25}$ GeV at which a non-inflationary solution is possible and show that the radiation dominated epoch alone technically is not able to explain the issue. An earlier paper on the subject, [1], attempts a multidimensional scenario. We review the scenario of the paper [1] in an effective four dimensional Einstein frame. We find that during contraction of extra dimensions, the residual bulk energy density is important and leads to a scaling solution. The existence of this scaling solution is a new result. We provide a numerical example which demonstrates the evolution of the scale factor and the extra dimensions. In the whole, the validity of the effective field theory calculations at the scale of $\Lambda = 10^{25}$ GeV is under question and, hence, the final conclusion regarding the possibility of a non-inflationary solution is preliminary.

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I. INTRODUCTION

One of the puzzles of the Big Bang Cosmology is the entropy problem. During the adiabatic expansion of the universe, the entropy per co-moving volume (S) is constant, and

$$S \propto g_* \alpha^3 T^3, \quad (1)$$

where g_* is the number of ultra-relativistic degrees of freedom, α is the scale factor and T is the temperature. g_* does not change by more than a couple orders of magnitude during the history of the universe and we neglect its time dependence in the following analysis. The entropy of the volume corresponding to the current Hubble patch, $H_0^{-1} \approx 10^{42} \text{ GeV}^{-1}$ [2, 3], which is stored in relativistic degrees of freedom is

$$S_U = \frac{8\pi^3}{135} g_* H_0^{-3} T_0^3 \simeq 10^{88}. \quad (2)$$

where 0 stands for current values. The value of S_U represents a puzzle. It takes its roots in the value of the present Hubble parameter, H_0 . The problem is called the entropy problem.

This puzzle together with the homogeneity of the observed universe [4] set the size problem. Going back to the Planck epoch one discovers that universe grew by a factor of

$$\frac{T_p}{T_0} = 10^{32}, \quad (3)$$

where $T_p = M_p = 1.22 \times 10^{19} \text{ GeV}$ is the Planck scale (in the following, p stands for Planck epoch values). The physical size at the Planck epoch corresponding to the current Hubble radius (H_0^{-1}) is $l_{H_0} = 10^{10} \text{ GeV}^{-1}$. In a power law expanding Universe the Hubble radius is roughly equal to the maximal causally connected region (horizon). However, at the Planck epoch the size of the universe corresponding to the Hubble radius at that time is $l_p = 10^{-19} \text{ GeV}^{-1}$. Hence, 10^{87} causally disconnected patches had to have approximately the same initial conditions in order that we observe homogeneous universe today [4]. This is the size problem which sometimes is called the homogeneity [5] or horizon [6] problem. Note that the number of patches (N) is

$$N = \frac{E_p}{M_p} = \frac{3}{4} S_U \quad (4)$$

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where $E = 3/4 S_U T$ is the energy of the relativistic degrees of freedom in the volume corresponding to the current Hubble patch.

The solution of the size/homogeneity/horizon problem is found if the presence of the large homogenous patch l_{H_0} with Planck energy density is explained. One way to achieve this is to blow up an initial patch of size $l_x = \Lambda^{-1} = 10^{-x} \text{ GeV}^{-1}$ to size l_{H_0} without significant loss of energy density. A period of accelerated expansion of the universe - inflation [7] is usually introduced for this purpose. Problems of inflationary cosmology [8] motivate attempts to find alternative explanations.

The paper [1] pretends to answer the size/homogeneity/horizon problem by a long period of intermediate evolution in a $d+4$ extra-dimensional universe. This allows slow enough decrease of the initial energy density without going into an inflationary regime. The scenario of the extra dimensional universe which is developed in the paper [1] has similar initial conditions to which is assumed in the hot Big Bang Cosmology, the only difference is number of dimensions. Namely, existence of a higher dimensional universe of linear size (Λ^{-1}) and energy density $\rho \sim \Lambda^{4+d}$ is assumed. To follow the evolution of the universe we assume that its content can be described by a perfect fluid and a weak anisotropic potential V . Initially, the evolution is governed by the perfect fluid (bulk matter). Isotropic expansion ends when V cannot be neglected any more. Then anisotropic evolution begins. The ultimate requirement on V is to prevent the decompactification of the extra dimensions. Once the energy density is dominated by the potential, the extra dimensions shrink while our dimensions continue to expand. Late time four dimensional cosmology is recovered once the extra dimensions are stabilized.

In this paper we show that the minimal value for Λ is 10^{25} GeV . The analysis is based on the fact that the universe has to achieve the current size (H_0^{-1}) . General arguments of the Section II show that 3+1 dimensional power law expanding universes with the one scale initial conditions (universe of linear size (Λ^{-1}) and energy density $\rho \sim \Lambda^4$) have to have $\Lambda \geq 10^{25} \text{ GeV}$ if no accelerated expansion is assumed, and for all Λ , a radiation dominated epoch alone leaves the size/homogeneity/horizon problem unresolved. Note, in general the radiation dominated epoch can explain the size/homogeneity/horizon problem if the initial energy density is assumed to be sufficiently high and there is no demand of the one scale initial conditions [11]. In the Section III, the scenario discussed in [1] is described from the effective four dimensional point of view and potentially can fit other extra-dimensional constructions. [17] Hence, we do not inter into details of the stabilization stage. We find analytical approximations for the evolution of the scale factor and the extra dimensions. In particular, we find a scaling solution which is in effect when two fluid components (bulk matter and a potential) cannot be neglected. The new result shows that whether expansion is inflationary depends not only on the slope of the potential but also on the number of the extra dimensions (d) and the equation of state of the bulk fluid (w). We provide a numerical example for demonstration purposes.

II. GENERAL ARGUMENTS

Let us start with an isotropic patch of the size $\Lambda^{-1} = 10^{-x} \text{ GeV}^{-1}$ and an initial energy density of order Λ^4 . These initial conditions are referred in the following as one scale initial conditions. To reach the size of the currently visible patch, $H_0^{-1} \approx 10^{42} \text{ GeV}^{-1}$, it has to expand by $x+42$ orders of magnitude. The evolution of the maximally symmetric hyper-surface without cosmological constant is described by Friedmann-Robertson-Walker (FRW) equations

$$3m_p^2 H_E^2 = \tilde{\rho} \quad (5)$$

$$-2m_p^2 \dot{H}_E = \tilde{\rho} + \tilde{p}, \quad (6)$$

where $m_p = 2.43 \times 10^{18} \text{ GeV}$ is the reduced Planck mass. The notations are as follows $\tilde{\rho}$ is the energy density, \tilde{p} is the pressure, α is the scale factor and $H_E = \dot{\alpha}/\alpha$ is the Hubble parameter in the Einstein frame. In a power law expanding universe $\alpha \propto t^k$ with $k > 0$, $H_E \propto t^{-1}$, $\tilde{\rho} \propto t^{-2}$ and, $\tilde{\rho} \propto \tilde{p}$. At the time of the Big Bang Nucleosynthesis ($\tilde{\rho}_r \approx \text{MeV}^4$) and afterwards the content of the universe is known to a high precision and so is the value of k . See e.g. [10]. During this period the universe has expanded by 11 – 12 orders of magnitude. Up to the time of the Big Bang Nucleosynthesis

$$\frac{\alpha_r}{\alpha_i} = \left(\frac{\tilde{\rho}_i}{\tilde{\rho}_r} \right)^{k/2}, \quad (7)$$

where i stands for initial and r - for the final values. For an arbitrary k , α grows by a factor of $10^{2k(x+3)}$

$$\frac{\alpha_r}{\alpha_i} = \left(\frac{10^x \text{ GeV}}{10^{-3} \text{ GeV}} \right)^{2k} = 10^{2k(x+3)}. \quad (8)$$

The requirement that the scale factor grows $x+31$ orders of magnitude imposes $x > 25$ for $k < 1$. Therefore, in a power law expanding universe, a non-inflationary solution of the size/homogeneity/horizon problem requires the

scale of initial energy density to be larger than $\Lambda = 10^{25} \text{ GeV}$. As this scale is much larger than the Planck scale, the FRW equations (6) can no longer be trusted. Note that in a radiation dominated universe the solution of the problem is technically impossible. Explicitly, in the radiation dominated universe ($k = 1/2$) the available $x + 3$ orders of magnitude are much smaller than the required $x + 31$ orders of magnitude. Hence, a new stage of evolution with a different equation of state is required. The arguments above are qualitatively different from the standard arguments like in [11] where there is no demand of the one scale initial conditions. In particular, a consequence of no demand of the one scale initial conditions allows to explain the size/homogeneity/horizon problem by the radiation dominated epoch alone.

III. CALCULATIONS IN THE EINSTEIN FRAME

We assume that the evolution of the $d + 4$ dimensional Universe is governed by the Einstein equations. Let G_{ab} be the metric for the full space-time with coordinates X^a . The line element of the spatially flat but anisotropic universe is

$$ds^2 = G_{ab} dX^a dX^b = -dt^2 + a(t)^2 d\mathbf{x}^2 + b(t)^2 d\mathbf{y}^2, \quad (9)$$

where \mathbf{x} denotes the three coordinates parallel to our visible three dimensional Universe and \mathbf{y} denotes the coordinates of the d perpendicular directions. The action of the Universe is described by

$$S = \int d^{d+4} X \sqrt{-\det G_{ab}} \left\{ \frac{1}{16\pi G_{d+4}} R_{d+4} + \hat{\mathcal{L}}_M \right\}, \quad (10)$$

where R_{d+4} is the $d+4$ dimensional Ricci scalar and $\hat{\mathcal{L}}_M$ is the matter Lagrangian density with the metric determinant factored out.

To follow the evolutions of 'our' three spatial dimensions from the effective four dimensional point of view, we replace the $b(t)$ by a canonically normalized scalar field $\varphi(t)$ which is related to $b(t)$ through

$$\varphi = \beta^{-1} m_p \ln(b), \quad (11)$$

where we have introduced

$$\beta^{-1} \equiv \sqrt{\frac{d(d+2)}{2}}. \quad (12)$$

To justify the use of the effective four dimensional treatment we need to make sure that 'our' three dimensions are large comparing to the extra dimensions. We also assume that all dimensions in the universe besides our 3 start small, of size $l = \Lambda^{-1}$, where Λ is the initial scale of the energy density. Furthermore, we assume that there is a patch in our three large dimensions of size $l = \Lambda^{-1}$ with the same energy density, and from now on we concentrate on the evolution of this patch. In terms of φ the effective reduced four dimensional action after performing a conformal transformation (with the help of [12]) to arrive at the Einstein frame is

$$S = \int d^4 x \sqrt{-\det \tilde{g}_{\mu\nu}} \left\{ \frac{1}{2} m_p^2 R_4 - \frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \varphi \tilde{\nabla}_\nu \varphi + \mathcal{V} e^{-d\varphi/m_p\beta} \hat{\mathcal{L}}_M \right\}, \quad (13)$$

where

$$\mathcal{V} = \int d^d \mathbf{y} = l^d \quad (14)$$

is the coordinate volume of the extra dimensions, and $\tilde{g}_{\mu\nu}$ is the 4d metric in the Einstein frame,

$$ds_E^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \alpha(t)^2 d\mathbf{x}^2. \quad (15)$$

The matter content (bulk matter and a potential) dictates the following Lagrangian [18]

$$\hat{\mathcal{L}}_M = -\rho - V(b) \quad (16)$$

where ρ is the energy density of the bulk matter and V is a potential responsible for the contraction of the extra dimensions. Potential V arises as a result of the construction of the space-time manifold. Specifically, in the paper [1], V is a confining potential between the orbifold fixed planes and has power law dependence on the scale factor b .

It is widely used that the part of the Lagrangian

$$\hat{\mathcal{L}}_{M1} = -\rho \quad (17)$$

provides the energy-momentum tensor for a perfect fluid, see e. g. [13, 14], while a derivation of the energy momentum from the action is left to a reader. In this paragraph we show the derivation. The energy-momentum tensor is obtained from variation of the action with respect to the metric

$$T_{ab} = -2 \frac{\delta \hat{\mathcal{L}}_{M1}}{\delta G^{ab}} + G_{ab} \hat{\mathcal{L}}_{M1}. \quad (18)$$

We can replace variation with respect to the metric with variation with respect to the determinant of the metric.

$$\delta \sqrt{-\det G_{ab}} = -\frac{1}{2} \sqrt{-\det G_{ab}} G_{ab} \delta G_{ab}. \quad (19)$$

In the case $\delta G_{ab} \neq 0$,

$$\frac{\delta \hat{\mathcal{L}}_{M1}}{\delta G^{ab}} = -\frac{1}{2} \sqrt{-\det G_{ab}} G_{ab} \frac{\delta \hat{\mathcal{L}}_{M1}}{\delta \sqrt{-\det G_{ab}}}. \quad (20)$$

However, the zeroth component of the metric is constant and, hence, $\delta G_{00} = 0$. To proceed let us define U_a to be the unit timelike vector orthogonal to the spatial slices, so

$$G_{ab} = -U_a U_b + \gamma_{ab} \quad (21)$$

with $U^a \gamma_{ab} = 0$. Therefore,

$$\frac{\delta \hat{\mathcal{L}}_{M1}}{\delta G^{ab}} = -\frac{1}{2} \sqrt{-\det G_{ab}} \gamma_{ab} \frac{\delta \hat{\mathcal{L}}_{M1}}{\delta \sqrt{-\det G_{ab}}}. \quad (22)$$

The higher dimensional energy-momentum tensor for a perfect fluid is supposed to take the familiar form

$$T_{ab} = (1 + w) \rho U_a U_b + w \rho G_{ab}, \quad (23)$$

where w is an equation of state parameter of the bulk matter. Moreover, we treat the case when w is a constant. The conservation of the higher dimensional energy-momentum tensor for a perfect fluid dictates

$$\rho \propto a^{-3(1+w)} b^{-d(1+w)}. \quad (24)$$

Variation of (24) with respect to $\sqrt{-\det G_{ab}} = a^3 b^d$ is equal to $-(1+w)\rho/(\sqrt{-\det G_{ab}})$. Plugging back all the above into (18), we obtain

$$T_{ab} = -\rho G_{ab} + (1+w)\rho \gamma_{ab}. \quad (25)$$

The familiar form of the energy momentum tensor for a perfect fluid (23) is recovered making use of the relation (21).

The effective 4 dimension Lagrangian is

$$\tilde{\mathcal{L}} = -\frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \varphi \tilde{\nabla}_\nu \varphi - \Omega - V(\varphi) \quad (26)$$

with

$$\Omega \equiv \mathcal{V} \rho e^{-d\beta\varphi/m_p} = \Omega_0 \alpha^{-3(1+w)} e^{-\gamma_2 \varphi} \quad (27)$$

$$V(\varphi) \equiv \mathcal{V} V(b) e^{-d\beta\varphi/m_p}, \quad (28)$$

where

$$\gamma_2 = \frac{d(1-w)\beta}{2m_p}. \quad (29)$$

In the above, Ω_0 is a constant and we made use of $\alpha = ab^{d/2}$.

The 4 dimensional energy-momentum tensor takes the form

$$\tilde{T}_{\mu\nu} = \tilde{\nabla}_\mu \varphi \tilde{\nabla}_\nu \varphi - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{g}^{\rho\sigma} (\tilde{\nabla}_\rho \varphi) (\tilde{\nabla}_\sigma \varphi) + (1+w) \Omega \tilde{\gamma}^{\mu\nu} - (\Omega + V(\varphi)) \tilde{g}_{\mu\nu}, \quad (30)$$

where

$$\tilde{g}_{\mu\nu} = -\tilde{U}_\mu \tilde{U}_\nu + \tilde{\gamma}_{\mu\nu}. \quad (31)$$

The equation of motion for the field φ is

$$\ddot{\varphi} + 3H_E \dot{\varphi} + \Omega' + V'(\varphi) = 0, \quad (32)$$

where $'$ denotes differentiation with respect to φ . Equation (32) together with the equation for the conservation of the energy momentum tensor

$$\tilde{\nabla}^\mu T_{\mu\nu} = \dot{\varphi} \ddot{\varphi} + V'(\varphi) \dot{\varphi} + \dot{\Omega} + 3H_E \dot{\varphi}^2 + 3H_E (1+w) \Omega = 0 \quad (33)$$

is consistent if

$$\Omega = \Omega(\varphi) \alpha^{-3(1+w)}, \quad (34)$$

which is indeed the case (see (24)). In the Einstein frame the energy density is $\tilde{\rho} = \tilde{T}_{00}$ and momentum is $\tilde{p} = \tilde{T}_{ii}/\tilde{g}_{ii}$ with $i = 1, 2$ or 3 . Then, the FRW equations read:

$$3m_p^2 H_E^2 = \tilde{T}_{00} = \left\{ \frac{1}{2} \dot{\varphi}^2 + V(\varphi) + \Omega \right\} \quad (35)$$

$$-2m_p^2 \dot{H}_E = \tilde{T}_{00} + \tilde{T}_{11}/\alpha^2 = \{ \dot{\varphi}^2 + (1+w) \Omega \}. \quad (36)$$

The three equations (27,35,36) fully determine the evolution of our three dimensions. Note that if the potential is positive the scale factor of our four dimensions, α , always grows since H_E^2 never reaches zero, hence, there is no collapse in the effective 4d Einstein frame. The effective equation of state in the Einstein frame for $-1 < w < 1$ is

$$1 \geq \frac{\tilde{p}}{\tilde{\rho}} = \frac{1/2 \dot{\varphi}^2 - V(\varphi) + w \Omega}{1/2 \dot{\varphi}^2 + V(\varphi) + \Omega} \geq -1. \quad (37)$$

Two forms for the potential have been considered in the literature: $V(\varphi) = M_0 e^{\gamma_1 \varphi}$ in [1] and $V(\varphi) = V_0 (1 - \zeta e^{-\gamma_1 \varphi})$ in [15]. The collapse of extra dimensions is possible only if $\gamma_1 > 0$. In the above M_0 , V_0 , ζ and γ_1 are constants which depend on the initial conditions. The second case $V(\varphi) = V_0 (1 - \zeta e^{-\gamma_1 \varphi})$ leads to inflation and is extensively treated in [16]. This is the case when inflation is free from the problem of initial conditions. In this paper, we concentrate on the case $V(\varphi) = M_0 e^{\gamma_1 \varphi}$ which has not been treated fully. In the following, we include the residual energy density of the bulk matter, the detail which has been neglected in [1], and calculate the evolution of the scale factor.

Let us divide the solution to different regions. In some regions the solutions for φ and α can be approximated by

$$\varphi = A_1 + A_2 \ln(t) \quad (38)$$

$$\alpha = A_3 t^k. \quad (39)$$

where A_1 , A_2 , A_3 and k are constants. Initially, when $\Omega > V$ the solution is

$$A_2 = \frac{2m_p^2 \gamma_2^2 + 3(1+w)^2 - 6(1+w)}{2\gamma_2(m_p^2 \gamma_2^2 - \frac{3}{2}(1+w)^2 - 3(1+w))} \quad (40)$$

$$\pm \frac{\sqrt{2m_p^2 \gamma_2^2 - 3(1+w)^2 - 16m_p^2 \gamma_2^2(m_p^2 \gamma_2^2 - \frac{3}{2}(1+w)^2 - 3(1+w))}}{2\gamma_2(m_p^2 \gamma_2^2 - \frac{3}{2}(1+w)^2 - 3(1+w))} \quad (41)$$

$$k = \frac{2 - \gamma_2 A_2}{3(1+w)} \quad (42)$$

$$A_2 = 3k A_2 = \gamma_2 \Omega_0 A_3^{-3(1+w)} e^{-\gamma_2 A_1}, \quad (43)$$

where A_1 and A_3 are determined by initial conditions and fit to later evolution. If $\dot{\varphi}$ dominates the evolution of the universe, the solution is the one of shift matter,

$$k = 1/3 \quad (44)$$

$$A_2 = \sqrt{\frac{2}{3}} m_p \quad (45)$$

Later on when the potential cannot be neglected any more, the description of the evolution deviates from the provided in [1]. Once the extra dimensions shrink, the solution becomes

$$k = \frac{2}{3} \frac{\gamma_1 + \gamma_2}{\gamma_1(1+w)} \quad (46)$$

$$e^{\gamma_1 A_1} = \frac{3m_p^2 k^2 \gamma_2 - \frac{1}{2} A_2^2 \gamma_2 + A_2 + 3k A_2}{M_0(\gamma_1 + \gamma_2)} \quad (47)$$

$$A_2 = -\frac{2}{\gamma_1}. \quad (48)$$

This is a scaling solution since the potential and the residual energy density in Ω dilute at the same rate. The solution is non-accelerating if $k < 1$. To see the difference from the conclusions of paper [1], recall that the solution, where only the potential is taken into account, is non-accelerating if $\gamma_1 \geq \sqrt{2}/m_p$ [9] (the slow roll parameter $\epsilon = m_p^2/2 (V'/V)^2 > 1$). Namely, k was determined entirely by γ_1 . In the present full analysis, we see that also γ_2 and w , i.e. properties related to the bulk fluid, are relevant for the determination of k . This difference allows one to find examples where non-inflationary potentials lead to inflation.

The following example demonstrates the above statement. Let us take the initial value for $\alpha_{in} = ab^{d/2} = 1$ and choose $\varphi_{in} = 0$. Further, we choose $\Lambda = 10^{-2} m_p$ and find $\Omega_0 = \Lambda^4 = 10^{-8} m_p^4$. The symmetry in the initial conditions allows to assume that \dot{a}/a and \dot{b}/b are of the same order. As a consequence, H_E^2 and $\dot{\varphi}^2$ in the equation (35) are of the same order. This permits us to estimate the initial value for $\dot{\varphi}(t_i) = 10^{-4} m_p^2$. Taking the scale of the potential $V(\varphi)$ to be $10^{-5} m_p$, we get $M_0 = 10^{-80} m_p^4$ and, hence, at the end of the contraction phase of the extra dimensions, the scale of the potential energy is higher the scale of the Big Bang Nucleosynthesis - 10 MeV. The choice of $d = 6$ and $w = -1/3$ gives $\gamma_2 = \sqrt{2/3}/m_p$. The choice of $\gamma_1 = \sqrt{8/3}/m_p > \sqrt{2}/m_p$ leads to a non-inflationary solution if only the potential is taken into account as in [1]. However, as noticed above, the scaling solution depends also on the values of γ_2 and w . Hence, as the calculation shows if the bulk fluid is taken into account ($k = 1.5$), there is acceleration. The numerical solution for the scale factor (see Fig. 1) supports this conclusion. To demonstrate the evolution of the extra dimensions, we solve numerically for φ (see Fig. 2).

IV. CONCLUSIONS

This paper discusses the entropy and size/homogeneity/horizon problems in power law expanding universes. We show that the radiation dominated stage alone in the universe with the one scale initial conditions cannot explain the issue regardless of the initial scale. A non-inflationary solution of the problems in the initially one scale universe is achieved only if this scale is allowed to be higher than 10^{25} GeV. At this energy scale, corrections to the FRW equations (6) should apply which are the subject of a followup work.

The extra-dimensional scenario developed in [1] is reconsidered in Section III. We perform the analysis of the evolution of a $d + 4$ dimensional universe in the effective Einstein frame. Since H_E is always positive and decreasing, the universe in the 4d Einstein frame never undergoes the stage of contraction. The earlier paper [1] neglects the bulk energy density in the investigation of the second stage of the evolution of the universe, the stage when the potential responsible for the contraction of the extra dimensions becomes important. Our new result is that the question whether the expansion is inflationary also involves the behavior of the bulk fluid. The numerical example explores the case where the potential itself does not cause inflation but the inclusion of the bulk energy density (the scaling solution) changes this.

The scenario of Section III basically describes a power law expanding universe. Throughout the evolution of the universe the power of the scale factor (k) changes. As shown in Section II, the minimal scale to resolve the size/homogeneity/horizon problem in power law expanding universe without inflation is $\Lambda = 10^{25} \text{ GeV}$. It is not generic at all, that such or higher energy density scale is allowed in a particular string theory model. In addition, the low energy field theory constructions should not be trusted at this or above scale since stringy effects become

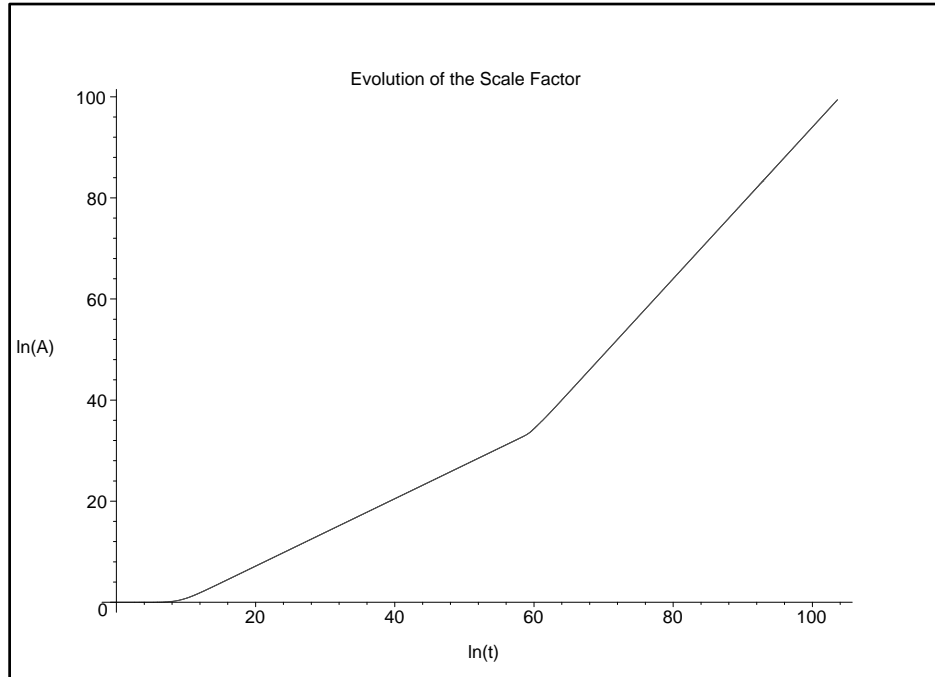


FIG. 1: The graph illustrates logarithm of the scale factor ($\ln(A)$) versus logarithm of time. While the extra dimensions expand (phase 1), our three dimensions decelerate $\ln(\alpha(t)) = 0.66 \ln(t) - 6$ and $k = 0.66$. Once the extra dimensions contract (phase 2), our dimensions accelerate $\ln(\alpha(t)) = 1.5 \ln(t) - 55$, namely $k = 1.5$ as expected from the scaling solution.

important. In this light the validity of the string theory based construction [1] to solve the problem is put under question.

Difficulty to find a non-inflationary solution based on this scenario might lead to interesting models of inflation (like in [15] and [16]). An interesting feature of the setup discussed here is the freedom from the problem of initial conditions on the onset of inflation, namely, no fine tuning of the slope of the potential. One starts with a dense unique scale patch filled with bulk matter and ends up with inflation. Note that in the cases [15] and [16], the energy density in the bulk matter during the second stage is rapidly diluted and does not have much influence on the behavior of the scale factor.

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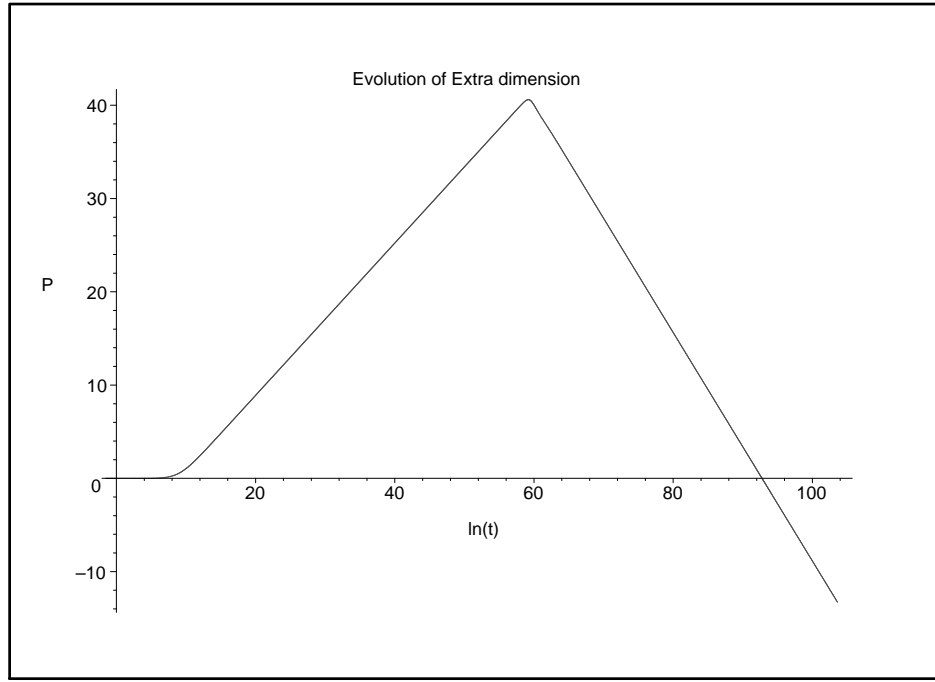


FIG. 2: The graph illustrates evolution of φ (P)- the size of the extra dimensions as function of time. Initially φ growing and then decreasing. The two phases correspond to the expansion and contraction phases of extra dimensions.

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